Voter model with zealots for opinion control and forecast of election results

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Papers

- A. Vendeville, B. Guedj & S. Zhou. Towards control of opinion diversity by introducing zealots into a polarised social group. *Complex Networks 2021.*
- A. Vendeville, B. Guedj & S. Zhou. Forecasting elections results via the voter model with stubborn nodes. *Applied Network Science* 6, 1 (2021).

Outline

1. The Voter Model with Zealots
2. Opinion Control under Backfire Effect
3. Predicting Elections Results

Original Model

Setting

- social network
- N users
- individual opinions in $\{0, 1\}$

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Dynamics

Wait $\Delta t \sim \mathrm{Exp}(N)$, then an user chosen uniformly at random adopts the opinion of a random neighbour of them. Repeat.

Long-time behaviour

Consensus [Aldous and Fill, 1994]

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Why?

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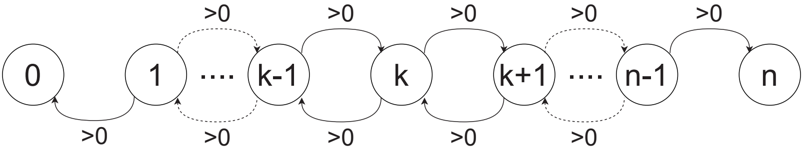
Why?

$N_1(t)$: quantity of opinon-1 users at time t. Absorbed in 0 and n.



Voter Model with Zealots

Framework

- Complete graph.
- $N_1(t)$: quantity of opinion-1 users at time t.
- (z_0, z_1) zealots.

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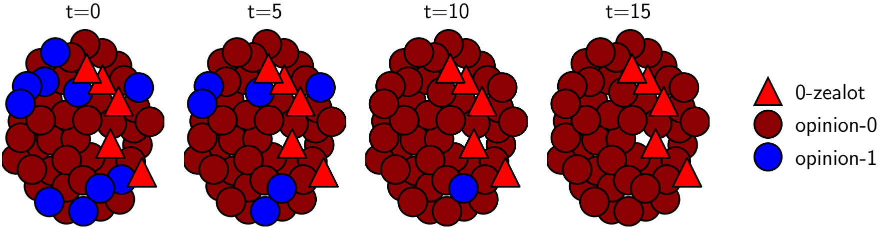
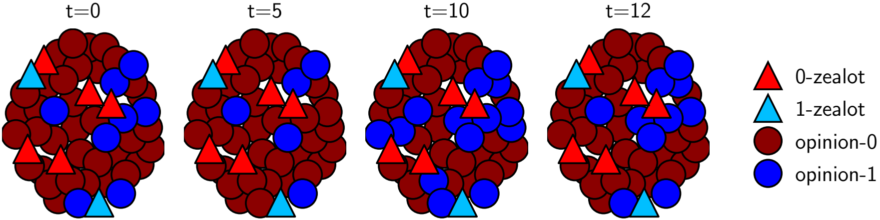
Long-time behaviour

1. $z_0 > 0, z_1 = 0 \Rightarrow$ almost surely everyone eventually has opinion 0.
2. $z_0 = 0, z_1 > 0 \Rightarrow$ almost surely everyone eventually has opinion 1.
3. $z_0 > 0, z_1 > 0 \Rightarrow$ equilibrium state.

We assume $z_0 + z_1 > 0$.

Example


t=0
t=5
t=10
t=15
0-zealot
opinion-0
opinion-1


t=0
t=5
t=10
t=12
0-zealot
1-zealot
opinion-0
opinion-1

Equilibrium (1)

Distribution of $N_1(t)$

At equilibrium,

$$N_1(t) \sim \text{Bin}\left(N, \frac{z_1}{z_0 + z_1}\right) \tag{1}$$

and thus

$$\mathbb{E}N_1(t) = N\frac{z_1}{z_0 + z_1}. \tag{2}$$

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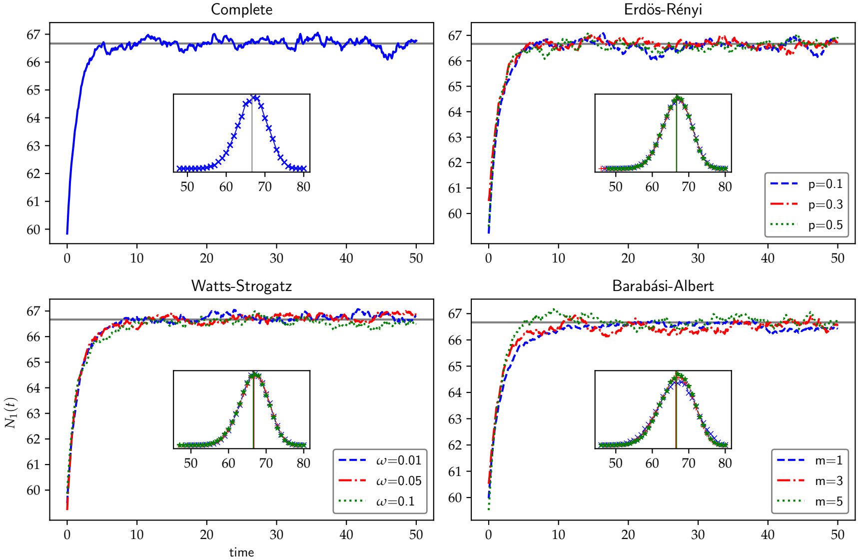
Average opinion of the society

We call $z_1/(z_0 + z_1)$ the **average equilibrium opinion.**

Equilibrium (2)

These results hold **on expectation** for any connected graph where zealots are placed uniformly at random.

Numerical simulations



$N = 100$, $z_0 = 20$, $z_1 = 40$. Averaged over 500 simulations.

- **Context:** network with $z_0 > 0$, $z_1 = 0$ (polarised community).
- **Goal:** get the average equilibrium opinion as close to a target $\lambda \in [0, 1]$ as possible.
- **Method:** we can inject zealots with opinion 1 into the network.

Problem Presentation (1)

- **Context:** network with $z_0 > 0$, $z_1 = 0$ (polarised community).
- **Goal:** get the average equilibrium opinion as close to a target $\lambda \in [0, 1]$ as possible.
- **Method:** we can inject zealots with opinion 1 into the network.

Backfire effect

The injection of $z_1^\star$ zealots with opinion 1 entices the radicalisation of $\alpha z_1^\star$ *free* users into zealots with opinion 0.

Problem Presentation (2)

Average opinion after backfire

$$f(z_0, z_1, \alpha) = \frac{z_1}{z_1 + \underbrace{(\alpha z_1 + z_0)}_{\substack{\text{updated } z_0 \\ \text{after backfire}}}} \tag{3}$$

Problem Presentation (2)

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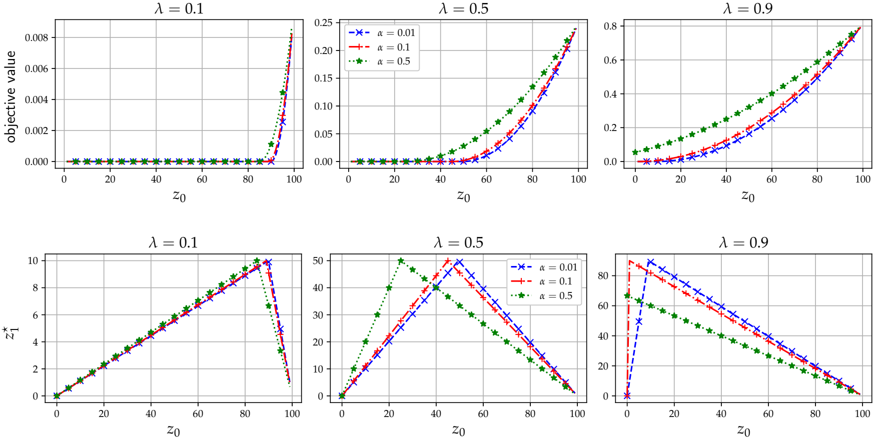
$$f(z_0, z_1, \alpha) = \frac{z_1}{z_1 + \underbrace{(\alpha z_1 + z_0)}_{\substack{\text{updated } z_0 \\ \text{after backfire}}}} \tag{3}$$

Optimisation problem

$$\underset{z_1}{\operatorname{argmin}} \quad |f(z_0, z_1, \alpha) - \lambda|^2 \tag{4}$$

$$\text{s.t.} \quad z_1 + \underbrace{(\alpha z_1 + z_0)}_{\substack{\text{updated } z_0 \\ \text{after backfire}}} \leqslant N \tag{5}$$

$N = 100$. Objective value **(top)** and optimal $z_1^\star$ **(bottom)** function of z_0.



Possible extensions

- More precise results for various graph structures
- Placement of zealots in the graph
- Optimisation with penalty for more flexibility
- Maximise magnetisation to encourage exposure to opposite views

Outline

Framework

Objective: predict the result of general elections (UK) and presidential elections (US) based on previous ones.

Method: assume result of previous elections are a realisation of the voter model with zealots, estimate parameters and predict future results.

Data: [UK] percentage of popular votes for Labour and Conservative parties in each general election since 1922.
[US] percentage of popular votes for Democratic and Republican parties in each presidential election since 1912.

Distribution over time

Assuming $s < t$ and a complete user graph,

$$\mathbb{P}\left(N_1(t) = k_t | N_1(s) = k_s\right) = \left[e^{(t-s)Q}\right]_{k_s,k_t} \tag{6}$$

where Q is the tridiagonal transition rate matrix with entries as below.

Entries of Q:

$$\begin{cases} q_{k,k-1} = (k - z_1)(N - k)/(N - 1) \\ q_{k,k+1} = k(N - k - z_0)/(N - 1) \\ q_{k,k} = -q_{k,k-1} - q_{k,k+1}. \end{cases} \tag{7}$$

Method

Choose $p \in \{\text{Labour, Conservative, Democratic, Republican}\}$.
x_i^p: percentage of votes for party p in the i^{th} election, rounded to the nearest integer.

How to predict x_i^p knowing $x_0^p, \ldots, x_{i-1}^p$?

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How to predict x_i^p knowing $x_0^p, \ldots, x_{i-1}^p$?

Forecast of election i

Assume $x_0^p, \ldots, x_{i-1}^p$ to be a realisation of the voter model with stubborn agents with $N = 100$ nodes. We estimate the values of the parameters (z_0, z_1) which gives us the theoretical distribution of x_i^p for the next election via (6).

A few remarks

- Predictions party by party.
- $x_0^p, \ldots, x_{i-1}^p$ [data] $\longleftrightarrow$ $N_1(0), \ldots, N_1(t_{i-1})$ [theory]
- z_1: guaranteed percentage of votes for party p
- z_0: guaranteed percentage of votes against p

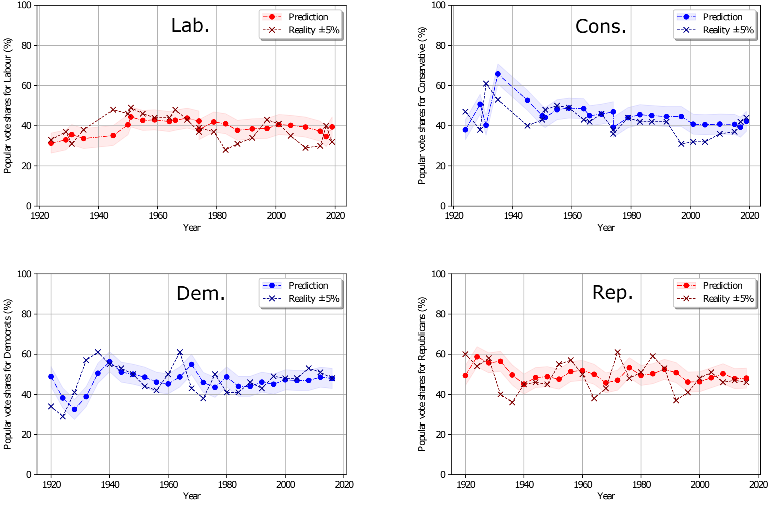
Maximum-likelihood for results up until election $i \geqslant 3$

$$(z_0^\star, z_1^\star) = \underset{z_0, z_1}{\operatorname{argmax}} \sum_{j=1}^{i-2} \log \left[e^{(t_{j+1}-t_j)Q}\right]_{x_j^p, x_{j+1}^p} \tag{8}$$

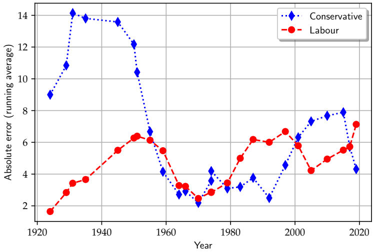
$\left[e^{(t_{j+1}-t_j)Q}\right]_{x_j^p, x_{j+1}^p}$: probability that the voter process with parameters (z_0, z_1) equals x_{j+1}^p at time t_{j+1} knowing it equaled x_j^p at time t_j.

Searches for parameters that maximise the probability of the (known) results for elections $1, \ldots, i-1$.

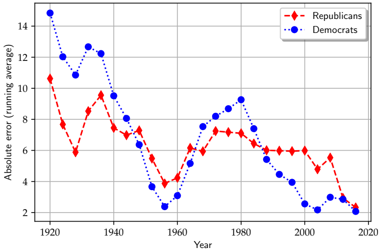
Predictions result



United Kingdom



United States



Discussion and possible extensions

- $(z_0^\star, z_1^\star)$ is valuable information on the political landscape
- Need more data (polls between elections?)
- Optimisation also on time parameter
- Equilibrium instead of dynamics analysis