Towards control of opinion diversity by introducing zealots into a polarised social group

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Contribution

In the context of the voter model with zealots, we

1. **derive a formula for the average opinion at equilibrium on any connected graph,**
2. **propose a strategy for increasing opinion diversity in a polarised social group.**

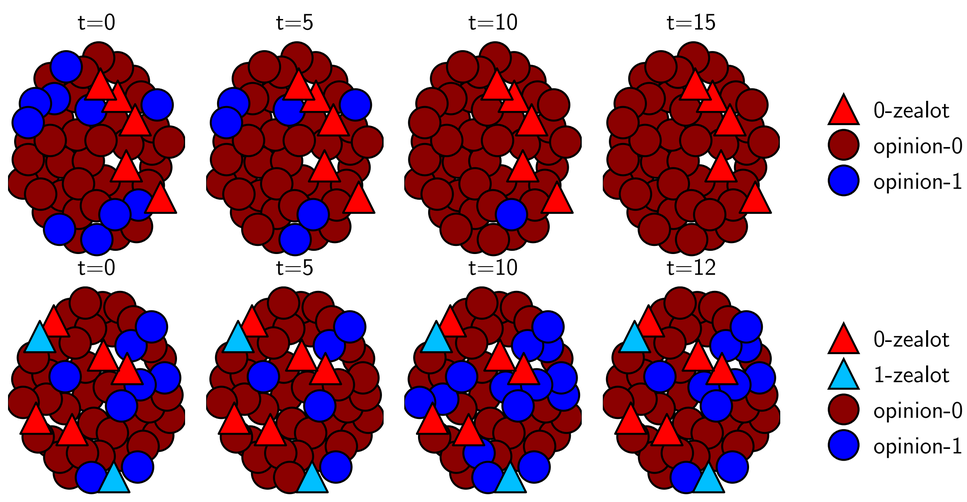
The Voter Model

1. Social graph with n users.
2. Initial opinions $x_1(0), \dots, x_n(0) \in \{0, 1\}$.
3. Repeat: wait a time $t \sim \text{Exp}(n)$ then a user chosen uniformly at random adopt the opinion of a neighbour chosen uniformly at random.

Assumptions

- Connected graph,
- z_0, z_1 zealots who never change opinion,
- zealots positions random uniform (we note $Z \sim \mathcal{U}$).

Notation $N_1(t)$: number of opinion-1 holders at time t.

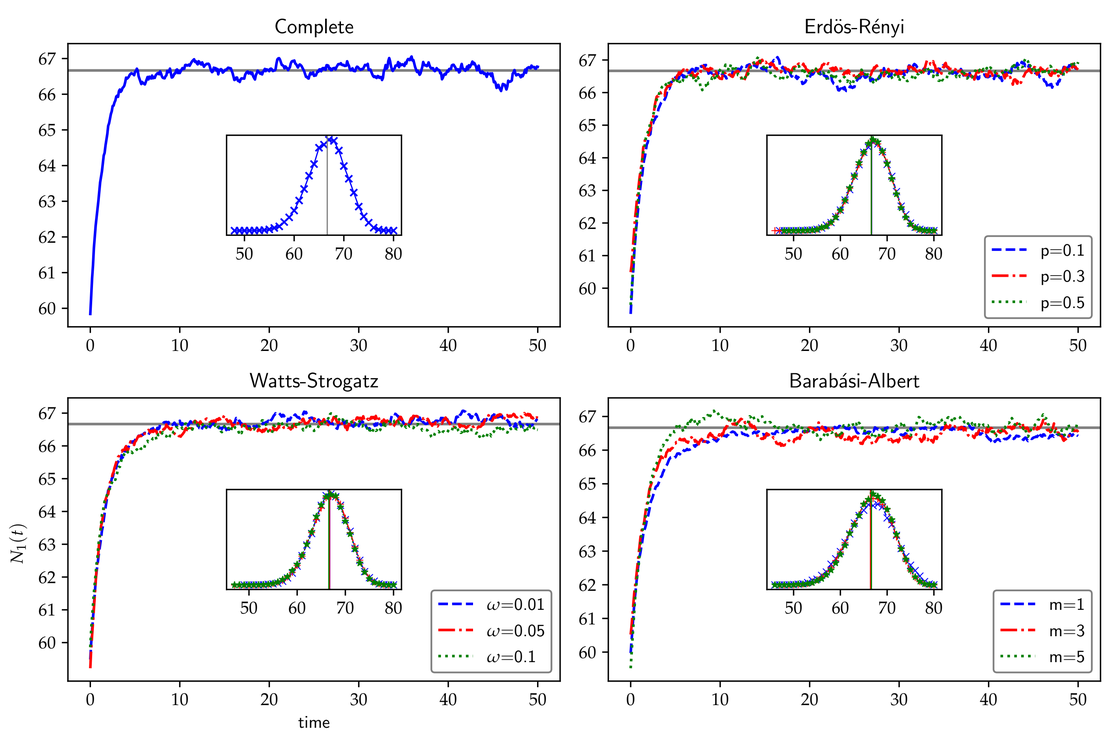


Example realisation on a complete graph. ***[Top]*** *no 1-zealots, everyone eventually adopts opinion 0.* ***[Bottom]*** *zealots in both camps, the system reaches a state of equilibrium where no opinion prevails.*

Equilibrium state

Theorem 1 *For any connected user graph and any z_0, z_1 such that $z_0 + z_1 > 0$, we have for large enough t*

$$\mathbb{E}_{Z\sim\mathcal{U}}\mathbb{E}N_1(t) = n\frac{z_1}{z_0 + z_1}. \tag{1}$$



Evolution of $N_1(t)$ with $n = 100$, $z_0 = 20$, $z_1 = 40$. Various graph models and parameters. Averaged over 500 simulations. Grey lines indicate limiting expectations from (1). Insets show empirical distributions at equilibrium. Nodes in the WS graph all have degree 4.

Control of opinion diversity

[Context] network with $z_0 > 0$, $z_1 = 0$ (polarised community), at equilibrium

[Goal] get the proportion of opinion-1 holders as close to a target $\lambda \in [0, 1]$ as possible,

[Method] we are able to sway the opinion of some of the non-zealots users to turn them into 1-zealots...

[Constraint] ... but if we sway z_1 nodes, a quantity αz_1 of non-zealots become 0-zealots (*backfire effect*).

Proportion of opinion-1 after backfire

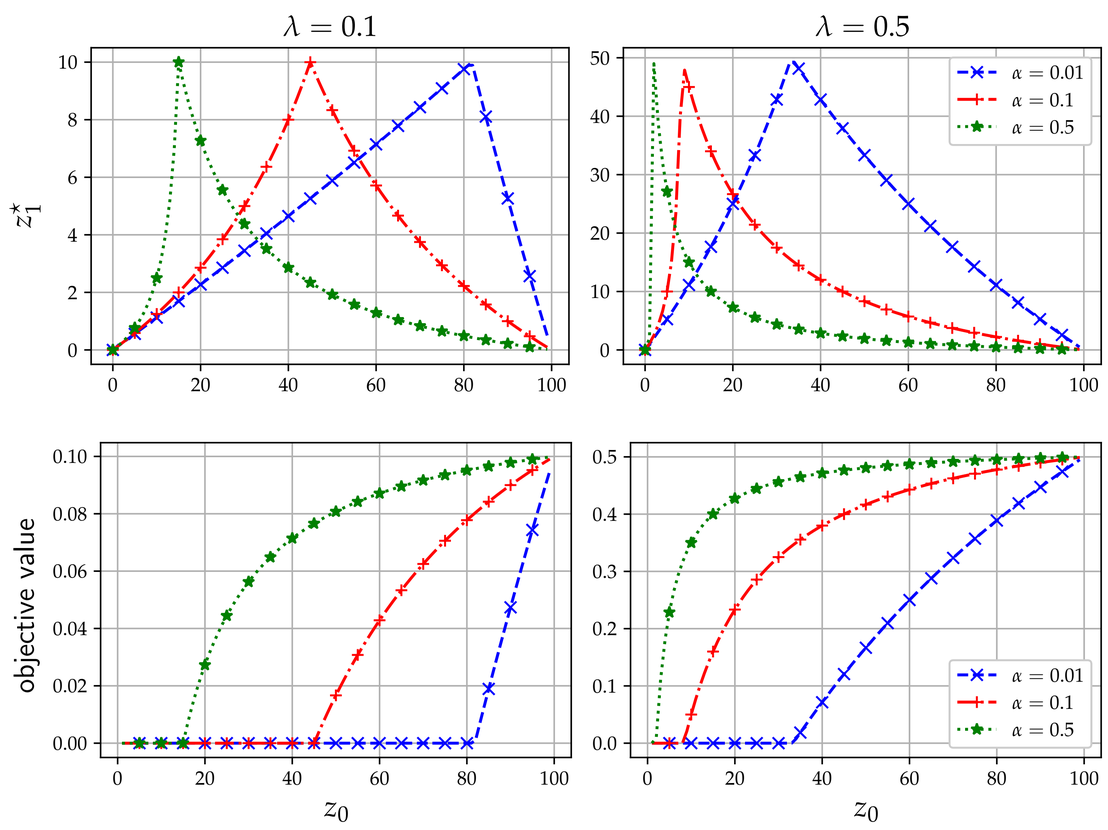
$$f(z_0, z_1, \alpha) = \frac{z_1}{(1 + \alpha z_1)z_0 + z_1} \tag{2}$$

Optimisation problem

$$\begin{aligned} &\underset{z_1}{\text{argmin}} \ |f(z_0, z_1, \alpha) - \lambda| \\ &\text{s.t. } z_1 + \underbrace{(1 + \alpha z_1)z_0}_{\substack{\text{updated } z_0 \\ \text{after backfire}}} \leq n \end{aligned} \tag{3}$$

Solution

$$z_1^\star = \begin{cases} \min\left(\dfrac{\lambda z_0}{1 - \lambda - \lambda\alpha z_0}, \dfrac{n - z_0}{1 + \alpha z_0}\right) & \text{if } \lambda < (1 + \alpha z0)^{-1} \\ \dfrac{n - z_0}{1 + \alpha z_0} & \text{otherwise.} \end{cases} \tag{4}$$



Control of opinion diversity for $n = 100$ and various z_0, α, λ. ***[Top]*** *Optimal $z_1^\star$ function of z_0. Before peaks $z_1^\star = \lambda z_0/(1 - \lambda - \lambda\alpha z_0)$, after peaks $z_1^\star = (n - z_0)(1 + \alpha z_0)$.* ***[Bottom]*** *Value of objective function $|f(z_0, z_1, \alpha) - \lambda|$.*

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